

Approximate number and approximate time discrimination each correlate with school math abilities in young children



Darko Odic^{a,*}, Juan Valle Lisboa^b, Robert Eisinger^c, Magdalena Gonzalez Olivera^b, Alejandro Maiche^b, Justin Halberda^c

^a University of British Columbia, Canada

^b Centro de Investigación Básica en Psicología, Uruguay

^c Johns Hopkins University, United States

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ABSTRACT

What is the relationship between our intuitive sense of number (e.g., when estimating how many marbles are in a jar), and our intuitive sense of other quantities, including time (e.g., when estimating how long it has been since we last ate breakfast)? Recent work in cognitive, developmental, comparative psychology, and computational neuroscience has suggested that our representations of approximate number, time, and spatial extent are fundamentally linked and constitute a “generalized magnitude system”. But, the shared behavioral and neural signatures between number, time, and space may alternatively be due to similar encoding and decision-making processes, rather than due to shared domain-general representations. In this study, we investigate the relationship between approximate number and time in a large sample of 6–8 year-old children in Uruguay by examining how individual differences in the precision of number and time estimation correlate with school mathematics performance. Over four testing days, each child completed an approximate number discrimination task, an approximate time discrimination task, a digit span task, and a large battery of symbolic math tests. We replicate previous reports showing that symbolic math abilities correlate with approximate number precision and extend those findings by showing that math abilities also correlate with approximate time precision. But, contrary to approximate number and time sharing common representations, we find that each of these dimensions uniquely correlates with formal math: approximate number correlates more strongly with formal math compared to time and continues to correlate with math even when precision in time and individual differences in working memory are controlled for. These results suggest that there are important differences in the mental representations of approximate number and approximate time and further clarify the relationship between quantity representations and mathematics.

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1. Introduction

What is the source of our intuitions about number? Recent work in cognitive development has focused on young children’s ability to quickly and intuitively represent the number of items in a collection through the Approximate Number System (ANS; Dehaene, 2009; Halberda & Feigenson, 2008; Halberda, Mazocco, & Feigenson, 2008; Halberda & Odic, 2014; Feigenson, Dehaene, & Spelke, 2004; Odic, Hock, & Halberda, 2014; Odic, Libertus, Feigenson, & Halberda, 2013). The multimodal ANS provides us with a rough and noisy sense of number, such as when guessing how many people are sitting in a lecture hall or how many items are in our shopping basket.

The ANS is characterized by three empirical signatures (Halberda & Odic, 2014; ; Feigenson et al., 2004). First, discrimination performance

in the ANS is ratio-dependent (i.e., obeys Weber’s law): discriminating a collection of 10 items from 5 items (a ratio of 2.0) is much easier than discriminating a collection of 10 items from 9 items (a ratio of 1.11). The precision with which an individual can successfully discriminate difficult ratios is often quantified through the Weber fraction (w), and theoretically corresponds to the amount of noise in the underlying ANS representations (Cordes, Gallistel, Gelman, & Latham, 2007; Halberda & Odic, 2014; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). Second, there are large individual differences in ANS precision, and children’s ANS continues to improve from birth onward, peaking around age 30 (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Odic, Libertus, Feigenson, & Halberda, 2013; Piazza et al., 2010). Finally, the ANS has been localized in both the human brain and in non-human animals to a region of the intraparietal sulcus (IPS; Dehaene, Piazza, Pinel, & Cohen, 2003; Nieder, 2005, 2012; Piazza et al., 2010; Roitman, Brannon, & Platt, 2007); physiological modulations of the IPS can, for example, enhance ANS discrimination (Cappelletti et al., 2013).

* Corresponding author at: Department of Psychology, University of British Columbia, 2136 West Mall, Vancouver, British Columbia V6T 1Z4, Canada.

E-mail address: darko.odic@psych.ubc.ca (D. Odic).

Researchers have also focused on the relationship between the ANS and formal mathematical abilities.¹ Individual differences in ANS precision show a small but significant relationship with formal math, including in preschoolers (Feigenson, Libertus, & Halberda, 2013; Libertus, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013) and adults (DeWind & Brannon, 2012; Libertus, Odic, & Halberda, 2012; Lyons & Beilock, 2011). Temporary modulations of the ANS can also selectively enhance or impair subsequent math performance (Hyde, Khanum, & Spelke, 2014; Park & Brannon, 2013; Wang, Odic, Halberda, & Feigenson, under review). Finally, individuals with math learning disabilities also show impaired ANS precision (Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010). This work, though not unchallenged (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013), suggests that our basic intuitions about math may emerge, in part, from a universal and ontologically ancient core cognitive system, and that intervening methods that improve the ANS may also help children in acquiring formal math concepts.

But the ANS is not alone in showing these behavioral and neural signatures. Many other dimensions, including surface area, time, density, weight, brightness, and line length, also obey Weber's law (Cantlon, Platt, & Brannon, 2009; Cheng, Srinivasan, & Zhang, 1999; Feigenson, 2007; Gescheider, 1997; Meck & Church, 1983; Möhring, Libertus, & Bertin, 2012; Stone & Bosley, 1965), develop with age (Brannon, Lutz, & Cordes, 2006; Droit-Volet, Clément, & Fayol, 2008; Odic, Le Corre, & Halberda, 2015; Odic et al., 2013), and are localized in the IPS (Cantlon et al., 2009; Castelli, Glaser, & Butterworth, 2006; Pinel, Piazza, Le Bihan, & Dehaene, 2004; Tudusciuc & Nieder, 2007). For example, transcranial noise stimulation of the IPS modulates both number and time discrimination (Cappelletti et al., 2013), and 6-month-old infant's Weber fractions for surface area discrimination appear identical to Weber fractions for number and time discrimination (Brannon et al., 2006; Feigenson, 2007). These similarities between distinct dimensions have led many researchers to suggest that number, time, and space are all represented by common mechanisms – a domain-general “generalized magnitude system” (Buetti & Walsh, 2009; Cantlon et al., 2009; Lourenco & Longo, 2010; Vicario, 2013; Walsh, 2003). Additional evidence for the generalized magnitude system comes from correlations of Weber fractions across dimensions (e.g., time and number; Meck & Church, 1983, but see Droit-Volet et al., 2008), and from persistent congruency and interference effects between quantities, whereby manipulation of one dimension affects discrimination performance of another (Barth, 2008; Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Gebuis & Reynvoet, 2012; Hurewitz, Papafragou, Gleitman, & Gelman, 2006; Leibovich & Henik, 2013; Lourenco & Longo, 2010; Szucs, Nobes, Devine, Gabriel, & Gebuis, 2013; Wood, Willmes, Nuerk, & Fischer, 2008).

Although researchers have frequently invoked the generalized magnitude system as an explanation for the behavioral and neural commonalities among quantity representations, it remains unclear what the shared mechanism between number and other dimensions might be. There are at least three (non-mutually exclusive) possibilities. First, quantity representations could share low-level sensory encoding processes. Dakin et al. (2011), for example, suggest that the number and density are both encoded through low spatial-frequency filters; hence, modulations of density (and thus of low spatial-frequency) will simultaneously impact number discrimination. Second, various dimensions might all be represented on an identical domain-general quantity scale and by identical sets of neurons that code for “more” or “less” of any and every dimension (Buetti & Walsh, 2009; Lourenco & Longo, 2010; Tudusciuc & Nieder, 2007); in this case, representations for, e.g., time and number, will show identical Weber fractions, identical

individual and developmental differences, and will be equivalently impacted by any modulation of the IPS. Finally, different dimensions may share common decision making or comparison computations, such as determining a threshold before the response is initiated; as a result, quantity representations may compete for behavioral responses and interfere with one another (DeWind & Brannon, 2012; Hurewitz et al., 2006; Van Opstal, Gevers, De Moor, & Verguts, 2008) and bottlenecks on attentional, memory, or decision making processes may result in similar Weber fractions across dimensions. Droit-Volet et al. (2008), for example, find that time and number Weber fractions only correlate when both dimensions are presented sequentially, suggesting that attentional and memory processes may be responsible for their correlation.

The existing evidence has not determined the best explanation for the common behavioral and neural signatures between number, time, and space (though most researchers seem to prefer the shared representations account). Recently, an increasing number of studies have attempted to dissociate quantity representations by examining how they relate to other cognitive abilities, such as affect (Droit-Volet, 2013; Young & Cordes, 2013) or formal mathematics (DeWind & Brannon, 2012; Lourenco, Bonny, Fernandez, & Rao, 2012). If, for example, the ANS correlates with formal math independently from non-numeric dimensions such as surface area, then we would have evidence for an important degree of independence between these dimensions. DeWind and Brannon (2012) recently found that while number and line-length discrimination correlate in precision in adults, only number correlates with formal math (as assessed by SAT scores). Similarly, Lourenco et al. (2012) found that while number and cumulative surface area correlate in precision amongst adults, individual differences in the ANS uniquely correlate with arithmetic math problems, while individual differences in cumulative area precision uniquely correlate with geometric math problems. Combined, this work suggests important distinctions in the representations of number and spatial extent and their relationship to formal math, and further implies that the commonality between these dimensions is unlikely to be due to both number and spatial extent being represented on an identical scale.

A similar kind of approach has been used to differentiate the ANS from approximate time perception. Time perception provides a useful case-study because its relationship to the ANS is still very actively debated. Meck and Church (1983) famously proposed that both time and number are encoded by an accumulating pacemaker mechanism, and found that amphetamine administration equally affects time and number perception in rats. Furthermore, affective stimuli, such as sad or happy faces, appear to impact both time and number equally (Droit-Volet, 2013), there are known shared neural substrates for time and number perception (Dormal, Dormal, Joassin, & Pesenti, 2012), and these dimensions show mapping and interference effects (Buetti & Walsh, 2009; Müller & Schwarz, 2008; Oliveri et al., 2008). Focusing on children and adults with math learning disabilities, previous work has shown mixed results in dissociating time from number perception. For example, Cappelletti, Freeman, and Butterworth (2011) find that time perception is not affected in adults diagnosed with dyscalculia. On the other hand, both Hurks and Loosbroek (2012) and Vicario, Rappo, Pepi, Pavan, and Martino (2012) find that children with math learning disabilities show abnormal time estimation and production. Combined, the existing work does not conclusively show evidence for or against time and number being part of a single generalized magnitude system.

The existing work on time, number, and their relationship to formal mathematics leaves open the possibility that quantity representations diverge and differentiate with development, especially as children acquire formal math concepts from preschool onward. Additionally, previous work has only tested children with math learning disabilities and used small sample sizes. Here, we examine the relationship between the ANS, time perception, and a series of formal math tests in a large sample of children tested at schools in Uruguay. By examining the

¹ In this paper, we refer to “formal” math in the sense of symbolic, abstract, school-taught mathematics, rather than differentiating between more informal math skills, such as addition and counting, and more formal math skills, such as word problems (e.g., see Libertus et al., 2013).

relationship between ANS, time discrimination, and math performance, we ask whether time and number share a common relationship with formal mathematics performance or two distinct relationships. If time and number pattern together, this would be strong evidence for them sharing a common representational resource. If they pattern separately (e.g., if number correlates more strongly with formal math and continues to correlate even after time performance is controlled for), this would be strong evidence for time and number relying on distinct representations. In short, by investigating the relationships between approximate number, approximate time and formal mathematics we can gain insight into the representational resources that support each of these abilities.

2. Methods

2.1. Participants

The present study was performed as part of Plan Ceibal, a Uruguayan initiative whereby each student in the public education system receives a technological device (e.g., computer and Tablet, etc.) for use in the classroom. The plan promotes digital inclusion and creativity in learning. We had access to 10 different schools and 31 different classrooms in Montevideo, the Uruguayan capital and its largest city. Within each classroom, each child received a tablet (described below) and, on the days that they were in the classroom and willing, participated in up to ten days of games and/or assessments. For the purposes of this paper, we only report data from the first four days (data from other days will appear in a series of other publications that address questions beyond number and time).

In total, we tested 503 unique first graders (Mean Age = 7.25; SD = 0.46; Age Range = 6.42–8.76). However, due to the opportunistic sampling, not all children completed all of the games. We include in our current sample only children who completed both the ANS and time discrimination games (N = 244; Mean Age = 7.26; SD = 0.47; Age Range = 6.42–8.71; 135 girls and 109 boys); analyses reported in the Results section show no significant differences between children who completed and did not complete both of these two games. Table 1 reports the Ns across the different formal math games that were used for all our statistical analyses. All children spoke Spanish as their first language and all tasks were administered in Spanish.

2.2. General procedure and apparatus

All testing was done in the child's classroom by trained researchers who followed a written protocol. In all, we tested children on seven games across four days: the Prueba Uruguaya de Matemática (PUMA)

(days 1 and 2), time discrimination (day 1), digit span (day 2), timed arithmetic (day 3), symbolic magnitude judgment (day 3), symbolic ordinal judgment (day 3), and the ANS discrimination game (day 4). These tasks are described in more detail below.

The games were played online in Spanish on a 7.6" × 4.65" × 0.39" XO Tablet running the Android 4.2 (Jelly Bean) with a 7" screen size. XO Tablets are hand-held, touch-responsive Tablets specifically designed for children aged 4–14 (see Fig. 1). Each participant had their own XO Tablet that they used throughout the duration of testing. All games were completely computerized and were developed using JavaScript, PHP, SQL, and JQuery. For games that involved sound, we provided each child with headphones to wear.

The Tablet was locked with game-specific passwords that children could not begin playing without the trained researcher informing them of the password. On the day of testing, the researcher would read a pre-written script explaining to children the games for that day and encouraging them to do their best. Games were explained on a blackboard to the whole class. Then, for each game, children were instructed to type in a specific password that would trigger the game. In this way, each game stayed paused until the researcher fully read the instructions and explained the game on the blackboard, assuring that children would not be playing the games early. While children played, the researcher walked around the classroom and provided any assistance.

2.3. ANS discrimination (Panamath)

Children's ANS precision was assessed via the standardized Panamath task (Halberda & Ly, 2015) on the fourth day. As illustrated in Fig. 1, children were shown two empty rectangles (a yellow one on the left, and a blue one on the right); subsequently, yellow dots appeared in the yellow rectangle and blue dots in the blue rectangle. The dots disappeared after 1600 ms. Children had to decide which side had more dots and indicated their response by tapping on the appropriate rectangle. We presented five ratios: 1.17 (e.g., 7 blue vs. 6 yellow dots), 1.2, 1.5, 2.0, and 3.0. The number of dots within each rectangle was always between 4 and 24. To control for surface area, half of the trials had cumulative surface area congruent with the number of dots (i.e., cumulative area and number gave identical answers) and half the trials were incongruent with the number of dots (i.e., cumulative area gave the opposite answer to that of number).

The first three trials acted as practice and were very easy. Children were allowed to play for 6 min. During the task, children were given feedback (a positive "ding!" sound for correct or an "err!" sound for incorrect answers). The dependent variable was percent correct across all completed trials.

Table 1

The pairwise correlations between all the tested variables. Stars indicate significance at $p < .05$.

	Discrimination tasks		Math tasks			Working memory	
	ANS	Time	PUMA	Timed arithmetic	Symbolic magnitude	Symbolic ordinal	Digit span
ANS (n = 244)	1.0	.26*	.39*	.37*	.51*	.44*	.21*
Time (n = 244)	–	1.0	.31*	.25*	.22*	.14	.29*
PUMA (n = 233)	–	–	1.0	.57*	.39*	.49*	.35*
Timed arithmetic (n = 153)	–	–	–	1.0	.44*	.47*	.35*
Symbolic magnitude (n = 170)	–	–	–	–	1.0	.46*	.22*
Symbolic ordinal (n = 119)	–	–	–	–	–	1.0	.37*
Digit span (n = 183)	–	–	–	–	–	–	1.0



Fig. 1. An illustration of the XO Tablet used for all the seven tasks, the ANS discrimination task, the time discrimination task, and the digit span task. In the time discrimination task, the two monsters would open their mouth and make a signing sound. In the digit span task, the child would have to re-create the set-size (here, set-size two is shown) after the numbers disappear.

2.4. Time discrimination

Children's time precision was assessed via the time discrimination game, which children played on the first day. As illustrated in Fig. 1, children were shown two monsters on a screen: a green one on the left and a purple one on the right. On each trial, each monster would take its turn making a singing-like sound for a certain amount of time. To provide additional visual cues, each monster would open its mouth while singing and put its hand over its mouth for a second once it was done. Children had to touch the monster that sang longer.

The first five trials acted as practice and were very easy. Children were presented with eight ratios: 1.20 (e.g., 1200 ms sound vs. 1000 ms sound), 1.25, 1.50, 1.60, 2.00, 2.40, 2.50, and 3.00. On half the trials, the left monster made sounds first. The singing sound also varied from trial to trial. Children were given feedback (a positive "ding!" sound for correct or an "err!" sound for incorrect answers). The dependent variable was percent correct across all completed trials.

2.5. Digit-span

To assess each child's working memory capacity, we administered a child-friendly version of the classic digit-span task (Baddeley, 1992). On each trial, children touched the screen to reveal between 1 and 5 cards (set-size) with a single-digit Arabic digit on each (see Fig. 1). The cards were shown simultaneously and stayed face-up on the screen for 1 s per card (i.e., three cards stayed face-up for 3 s, five cards for 5 s, etc.). After this time period, the cards turned face-down. Subsequently, blank cards appeared underneath the face-down cards and the child had to input the correct identity for each card in the correct order. Trials on which children got the numbers correct but in the wrong order were counted as incorrect. To determine each child's digit-span, we used a staircase method: each child started with set-size of 1, and the set-size increased by one every time the child got two trials correct; if the child got a trial wrong, the set-size would decrease by 1. The span is determined by the maximum set-size that the child successfully completed. The task was limited to 4 min.

2.6. Formal math assessment #1: Prueba Uruguaya de Matemática (PUMA)

The PUMA was used to test a broad set of math skills through a series of mini-games, including number symbol knowledge, Arabic number

ordering, number composition and decomposition, number line placement, and basic word problems. These were tested within the PUMA using eight different mini-games across two testing days. On each day, children were given up to 10 min total to complete as many questions (intermixed from various mini-games) as they could; if they spent more than 1 min on any single question, the game automatically advanced to the next one. All children did the trials in the same order. Children received instructions verbally in their headphones via the Tablet after typing in the game-specific password given them by the researcher.

Videos outlining each of the eight tasks can be found (in Spanish) online (<http://www.ceibal.psico.edu.uy/2013/preparacion/prueba-de-evaluacion-de-matematica/>). Examples of three of the mini-games are shown in Fig. 2.

In the first mini-game, children had a set of cards with numbers on the screen (1, 2, 5, 10, 20, 50, and 100). On each trial, the on-screen game character desired a target number (written on the screen), and children had to drag and add several cards to a workspace in order to match the target number (targets were 10, 8, 20, 34, 52, and 100). In the second mini-game children performed the reverse of this challenge – i.e., a game character on-screen showed a collection of cards (e.g., 50, 1, 1, 1) and the child had to select the correct number for the total array from among 3 options (e.g., 53). In the third mini-game ("base ten", Fig. 2a), children had to add cards (each with a value of 10) to match a target value (e.g., 100). Target values were 100, 60, 120, 80, 40, and 30. The fourth mini-game tested number line ordering (Fig. 2b). Children were shown a train with a number line and some values missing. Children then had to select a card with a number and drag it into the missing spot (the targets were 5, 8, 10, 12, 15, 18, and 20). The fifth mini-game tested verbal counting and cardinality. Children were shown a character and some dots and they had to count the number of dots. They then had to move the character along a line to match the number of the dots (targets were 16, 12, 20, and 9). The sixth mini-game tested number order. Children were shown numbers 10–1 in a scrambled order, and had to arrange the cards in decreasing order by dragging them from the bottom of the screen to their correct position on the top. In the seventh mini-game, children saw an empty number line with a single anchor number (either 5, 7, 6, 3, 5, or 8), and had to drag two numbers onto the appropriate position of the line given the anchor (e.g., place 4 and 9 on the line relative to the anchor of 5). Targets were 4–9, 1–9, 4–10, 5–9, 1–7, or 1–6. In the eighth mini-game, testing basic word problems, children saw toys

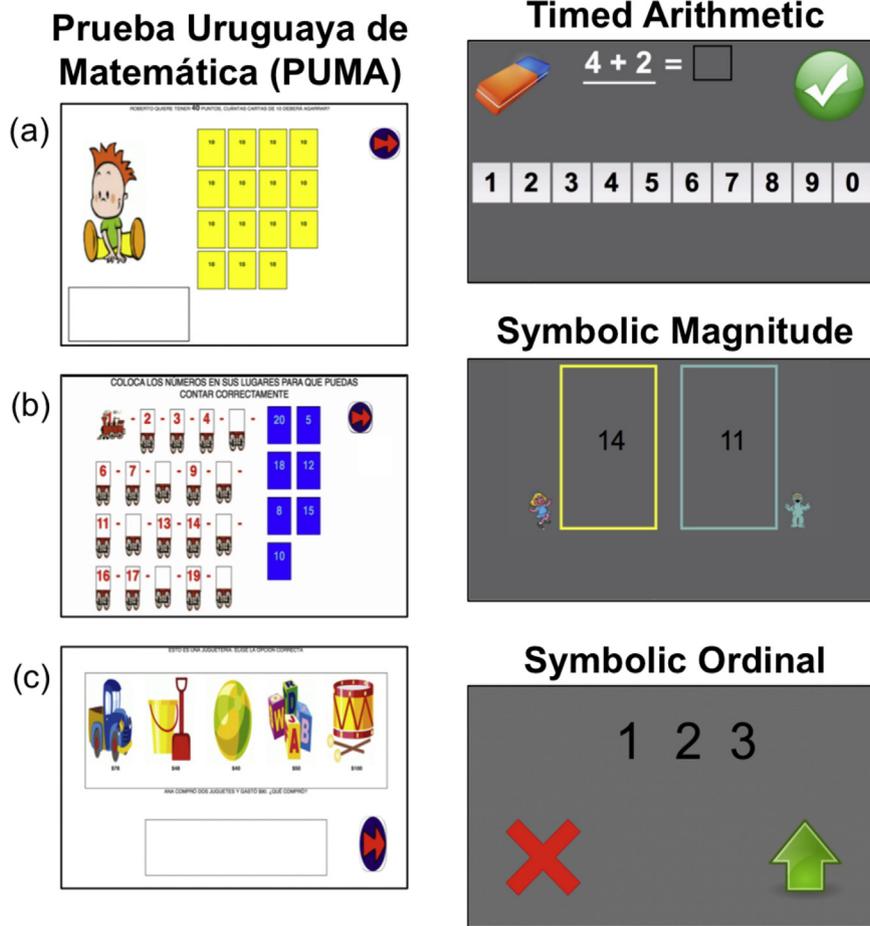


Fig. 2. An illustration of the four Formal Math tasks. The PUMA task is comprised of eight mini-games, of which three are shown on the left: (a) adding game, (b) number line ordering, and (c) word problems. Details on each of the eight games are in-text.

on the screen with labeled prices (Fig. 2c). Three math problems indicated the toys to select: (1) “Select two toys totalling \$90”. (2) “If the girl only has \$90, which toy can she not buy?” (3) “Matthew paid \$80 and had \$2 in change. What did he buy?”

The dependent measure was percent correct across all completed questions. The PUMA was designed as an overall assessment of math ability, and preliminary analyses revealed insufficient numbers of trials to investigate each game individually. As a result, we combined the percent correct across the eight mini-games; as shown in the Results section, this single score showed a good, normal distribution of scores.

2.7. Formal math assessment #2: Timed arithmetic

The timed arithmetic game was played the third day and was used to assess children's addition and subtraction abilities. As illustrated in Fig. 2, children were shown single Arabic digit subtraction and addition problems (e.g., $4 + 2 = ?$) and had to type their answer on a linear number pad. Problems always involved single-digit operations, but could include decade breaks (i.e., going above 10, 20, etc.). All children saw an identical order of trials. When done, children pressed a green checkmark to register the answer or an eraser to clear the answer and change it. The first two trials were considered practice. Children were given feedback (a positive “ding!” sound for correct or an “err!” sound for incorrect answers). No time limit was enforced for each arithmetic problem. The dependent variable was percent correct across all completed questions.

2.8. Formal math assessment #3: Symbolic magnitude judgment

The symbolic magnitude judgment task was also played on the third day and was used to assess children's knowledge of Arabic digits. As illustrated in Fig. 2, children were shown two digits (one on each side of the screen) within colored rectangles and had to tap on the one that was numerically larger. The lowest number presented was 1, and the highest number presented was 21. All children saw the same order of trials. Children were allowed to play for 4 min, but could spend as much time as they needed on each problem. The first two trials were considered practice. Children were given feedback (a positive “ding!” sound for correct or an “err!” sound for incorrect answers). The dependent variable was percent correct across all completed questions.

2.9. Formal math assessment #4: Symbolic ordinal judgment

The symbolic ordinal judgment game was also played on the third day and assessed children's knowledge of number order. As shown in Fig. 2, children were shown three single-digit numbers on the screen and had to decide if the numbers were in increasing order (e.g., 3–5–8) or not (e.g., 3–8–5). If the children thought the numbers were increasing, they had to press an upward facing green arrow; if they thought the numbers were not increasing they had to press a red X. The first three trials were considered practice. All children saw the identical order of trials. Children could play for up to 4 min, but as long as needed on each trial. Children were given feedback (a positive

“ding!” sound for correct or an “err!” sound for incorrect answers). The dependent variable was percent correct across all completed questions.

3. Results

Because all testing occurred in the school setting over four days, many children did not complete all seven of the tasks (i.e., ANS discrimination, time discrimination, digit span, and the four formal math assessments). Our sample sizes and pairwise correlations for each of the tasks are presented in Table 1.

To make sure that our attrition was random and not because some children were doing poorly on the games and subsequently dropped out, we first examined the scores of children who completed only the ANS discrimination (on day 4) or only the time discrimination game (on day 1) and compared them to children who completed both tasks. We found no significant difference in ANS discrimination performance between children who completed only the ANS discrimination game ($M = 0.73$, $SD = 0.11$, $N = 151$) and those who completed both the time discrimination and ANS discrimination games ($M = 0.74$, $SD = 0.09$, $N = 251$; $t(400) = -0.54$; $p = 0.59$). Similarly, there was no difference in time discrimination performance between children who completed only the time discrimination game ($M = 0.68$, $SD = 0.20$, $N = 162$) and those who completed both the ANS discrimination and time discrimination games ($M = 0.68$, $SD = 0.20$, $N = 251$; $t(411) = -0.12$; $p = 0.91$).

We next tested whether performance on both ANS and time discrimination obeyed Weber's law (i.e., was ratio-dependent). As shown in Fig. 3, a one-way Repeated-Measures ANOVA (Ratios: 1.17, 1.25, 1.50, 2.00, 3.00) over ANS discrimination percent correct showed a significant effect of Ratio ($F(4,972) = 347.82$, $p < .001$) and a normal distribution of scores. And, as also shown in Fig. 3, a one-way Repeated-Measures ANOVA (Ratios: 1.20, 1.25, 1.50, 1.60, 2.00, 2.40, 2.50, and 3.00) over time discrimination percent correct showed a main effect of Ratio ($F(7,1050) = 25.93$, $p < .001$), with a slightly left-skewed

distribution of scores. Additionally, we found that ANS discrimination performance ($M = 73.7\%$, $SD = 9.37\%$) was significantly better than time discrimination performance ($M = 68.83\%$, $SD = 9.37\%$; $t(243) = 3.89$; $p < .001$), consistent with previous work (Droit-Volet et al., 2008). We found no significant relationship between age and ANS discrimination ($r(233) = -.07$; $p = .29$) or between age and time discrimination ($r(233) = -.01$; $p = .87$), probably due to our truncated age range.

Consistent with previous work, we found a weak but significant correlation between ANS discrimination and time discrimination performance ($r(242) = .26$; $p < .001$; Fig. 3). As discussed in the Introduction, this correlation could be due to shared encoding, identical domain-general representations, or decision-making components. To further understand this correlation, we investigated the relationship of both ANS discrimination and time discrimination to individual differences in digit span – the classic measure of working memory performance. Digit span performance correlated with both ANS discrimination ($r(182) = .21$; $p < .01$), and with time discrimination performance ($r(182) = .29$; $p < .001$), suggesting that working memory contributes to individual differences in each task. But – consistent with the idea that number and time only share decision-making components – we failed to find a correlation between ANS and time discrimination when digit span was controlled for ($r(179) = .10$; $p = .16$; Fig. 3). In other words, the relationship between the ANS and time in our sample appears to be largely accounted for by shared working memory (see also Droit-Volet & Wearden, 2001; Genovesio, Tsujimoto, & Wise, 2012). However, as it is difficult to draw conclusions from null results, we next turn to the relationship between time, number and our formal math assessments.

Each of the individual math tasks ranged in performance from 0 to 100%, but chance performance was different for each task (e.g., for PUMA and timed arithmetic, chance was less than 5%, while for symbolic magnitude and symbolic ordinal was 50%). As shown in Fig. 4, most of the individual math tasks were normally distributed, with the exception of the symbolic magnitude task, which showed a strong negative skew

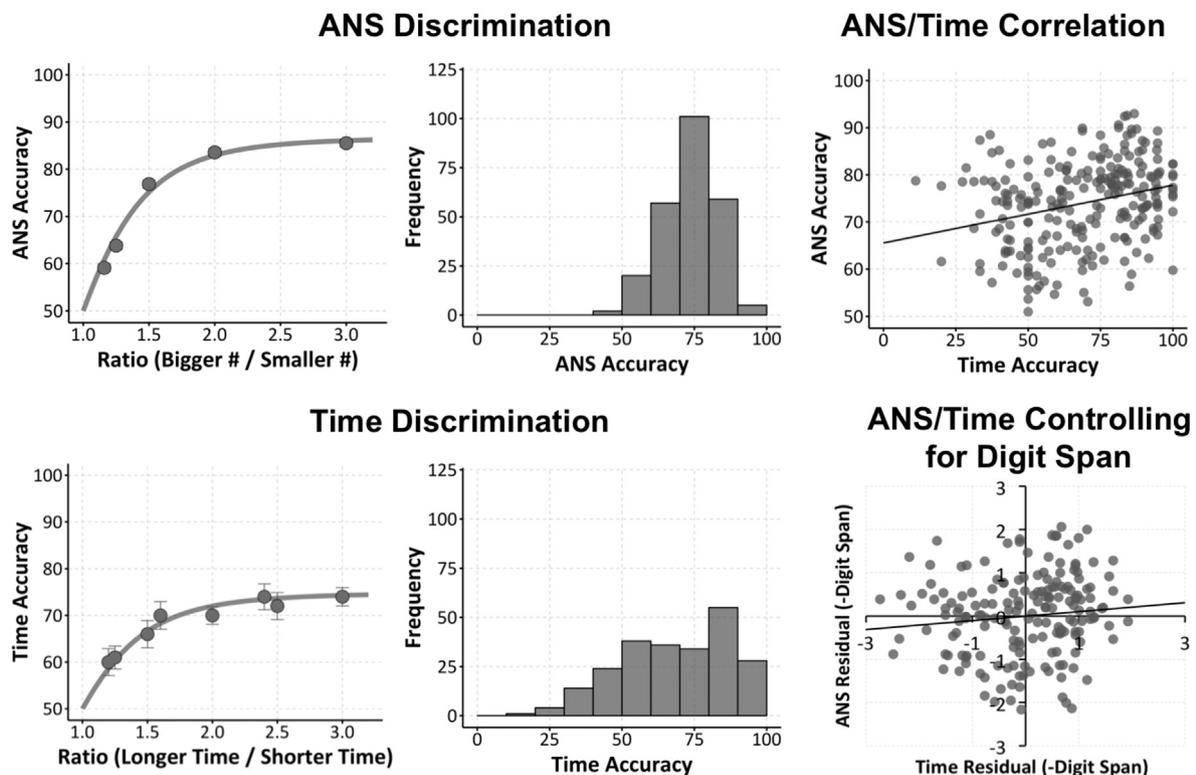


Fig. 3. Performance on the ANS discrimination and time discrimination tasks, averaged across all children and as a histogram. Results from both tasks show clear ratio-dependent effects and normal distributions of scores (the curve fits are from a psychophysical model of Halberda & Feigenson, 2008). On the right, the correlation between ANS and time performance, and the correlation between ANS and time when controlling for digit span.

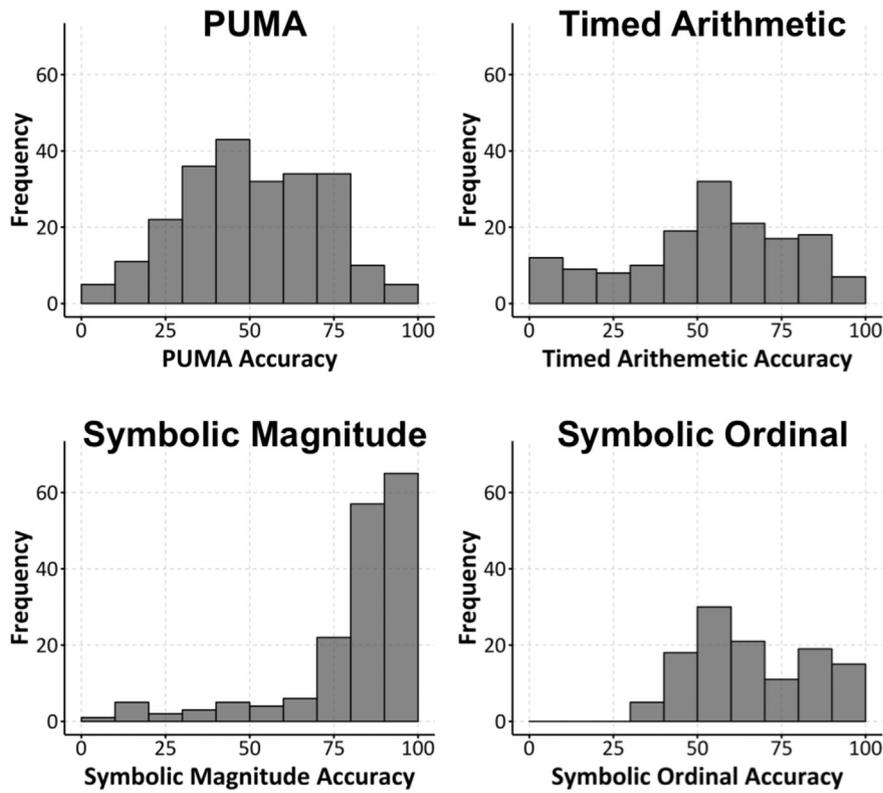


Fig. 4. Histograms for each of the four math tasks (PUMA, timed arithmetic, symbolic magnitude, and symbolic ordinal).

and was easier than the other three tasks. The average performance on the PUMA task was 50.5% (SE = 1.4%), on the timed arithmetic task 53.1% (SE = 2.0%), on the symbolic magnitude task 80.8% (SE = 1.5%), and on the symbolic ordinal task 66.2% (SE = 1.6%). These distributions show that children understood and engaged with all of the math tasks.

Because of the strong correlations between the four math tasks and in order to maximize our sample size, we calculated a single combined Formal Math Score. To do so, we first normalized performance on each individual math task by re-computing each child's score as a Z-Score. Then, to calculate the combined and standardized Formal Math Score, we averaged these individual Z-Scores across all children, ignoring the scores for the tasks they did not complete. This gave us a final sample size of 244 individual children for whom we had a combined Formal Math Score, ANS discrimination and time discrimination. As shown in Fig. 5a, the Formal Math Score distribution was normal with a mean of -0.05 (SD = 0.88). We found no correlation between age and the Formal Math Score ($r(232) = -.03; p = .69$), probably due to our truncated age range.

Next, we turn to the main question of interest – what is the relationship between the ANS discrimination and the Formal Math Score and is

it in any way different from the relationship between time discrimination and the Formal Math Score? We found a moderate correlation between ANS discrimination performance and the Formal Math Score ($r(242) = .51; p < .001$; Fig. 5b) and a weak correlation between time discrimination performance and the Formal Math Score ($r(242) = .29; p < .001$; Fig. 5c). This relationship held for each of the individual math tests that comprised the combined Formal Math Score, with the exception of symbolic ordinal task, which did not correlate with time discrimination (see Table 2). We also found that the correlation between ANS discrimination and the Formal Math Score was significantly higher than the correlation between time discrimination and the Formal Math Score ($Z = 3.22; p < .001$). This difference could, at least in part, be due to the lower reliability of the time discrimination scores and does not by itself suggest independence between time and number.

To further understand these results, we performed a set of partial correlations; if time and number are both represented on a common scale by common Gaussian tuning curves, we should find that controlling for one should remove the correlation with the Formal Math Score. However, contrary to this, we found that ANS discrimination performance still correlated with the Formal Math Score, even when the

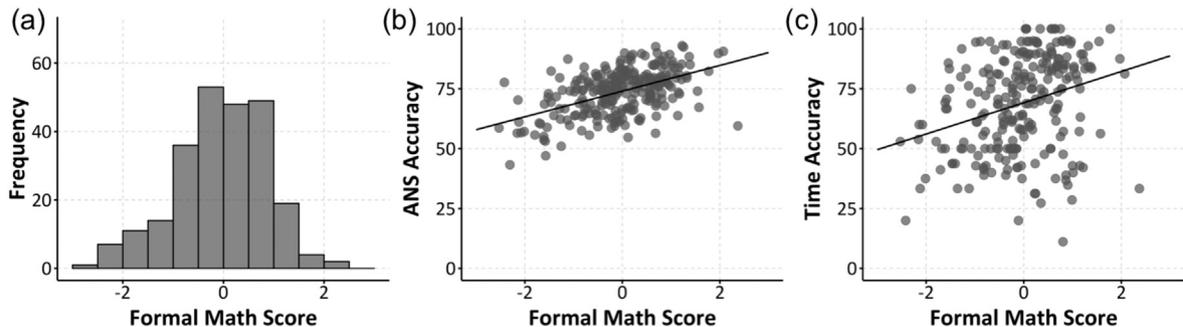


Fig. 5. (a) Histogram of the combined Formal Math Score; (b) The correlation between the Formal Math Score and the ANS task; (c) The correlation between the Formal Math Score and the time discrimination task.

Table 2
The data from the different tasks, alongside each task's correlation with ANS discrimination performance, with the ANS discrimination performance when controlling for time discrimination, correlation with time discrimination performance, and correlation with time discrimination performance when controlling for ANS discrimination. Stars indicate significance ($p < .05$).

	N	Average (SE)	ANS discrimination		Time discrimination	
			Correlation	Controlling for time	Correlation	Controlling for ANS
ANS discrimination	244	73.4 (0.6)	–	–	.26*	–
Time discrimination	244	68.8 (1.3)	.26*	–	–	–
Formal Math Score	244	–0.05 (0.06)	.51*	.47*	.29*	.19*
PUMA	233	50.5 (1.4)	.39*	.33*	.31*	.23*
Timed arithmetic	153	53.1 (2.0)	.37*	.34*	.25*	.18*
Symbolic magnitude	170	80.8 (1.5)	.51*	.49*	.22*	.14
Symbolic ordinal	119	66.2 (1.6)	.44*	.42*	.14	.08

variability in time discrimination was entirely controlled for ($r(242) = .47$; $p < .001$). We also found the converse – time discrimination performance still correlated with the Formal Math Score, even when the variability in ANS discrimination was entirely controlled for ($r(242) = .19$; $p < .05$). These results held true for all of the individual formal math games, with the exception that symbolic magnitude no longer correlated with time discrimination when ANS was controlled for (see Table 2). Hence, each dimension has a unique correlation with Formal Math Score.

A potential criticism, however, is that the differences captured by the partial correlations between ANS, time, and formal math tasks are not due to independent representations of time and number, but due to differences between the two discrimination tasks. Specifically, because stimuli in the time discrimination task are, unlike the ANS discrimination task, presented sequentially, performance in this task will highly depend on individual differences in working memory (Droit-Volet & Wearden, 2001). Thus, a correlation between time discrimination and the Formal Math Score could remain significant even if the ANS forms a domain-general magnitude system with time, since the leftover variance could be due to different working memory demands of the sequential vs. simultaneous tasks.

To investigate the possibility that working memory can account for the correlation between time discrimination and the Formal Math Score, we further investigated individual differences in the digit span task. If ANS and time comprise a generalized magnitude system and the significant partial correlation between time discrimination and the Formal Math Score is due to working memory demands, we should find that further controlling for digit span performance should eradicate this correlation. Contrary to this prediction, however, we found that time continued to weakly correlate with Formal Math even when both ANS and digit span were partialled out (Fig. 6a; $r(178) = .18$; $p < .05$).

Similarly, the ANS continued to moderately correlate with formal math even when both time and digit span were partialled out (Fig. 6b; $r(178) = .43$; $p < .001$). In other words, ANS and time uniquely and independently correlated with formal math performance, even when individual differences in working memory were accounted for.

4. General discussion

While previous work has largely focused on the commonalities between time and number discrimination, including shared encoding and similar Weber fractions, it has remained unclear just how similar these representations really are. Under many popular accounts of the generalized magnitude system, time and number are represented on an identical, domain-general scale that codes for “more” or “less” of any quantity; under extreme version of such a view, time and number should be near-perfectly correlated and show an identical relationship to other cognitive abilities, including formal mathematics. Contrary to this account, however, our findings show that the ANS and time perception do not correlate in preschoolers when individual differences in working memory are controlled for, and that ANS and time uniquely and independently correlate with school math abilities. Furthermore, the independence between ANS and time perception is unlikely to be caused by task-related differences (e.g., by time being presented sequentially), as ANS and time continue to correlate with formal math performance when working memory is controlled for.

Our results are most consistent with the idea that the scale that codes approximate number is distinct from the scale that codes for approximate time. In other words, while time and number might share encoding and decision-making components – including working memory demands – the evidence presented here suggests that they do not share identical, domain-general representations. Thus, our results

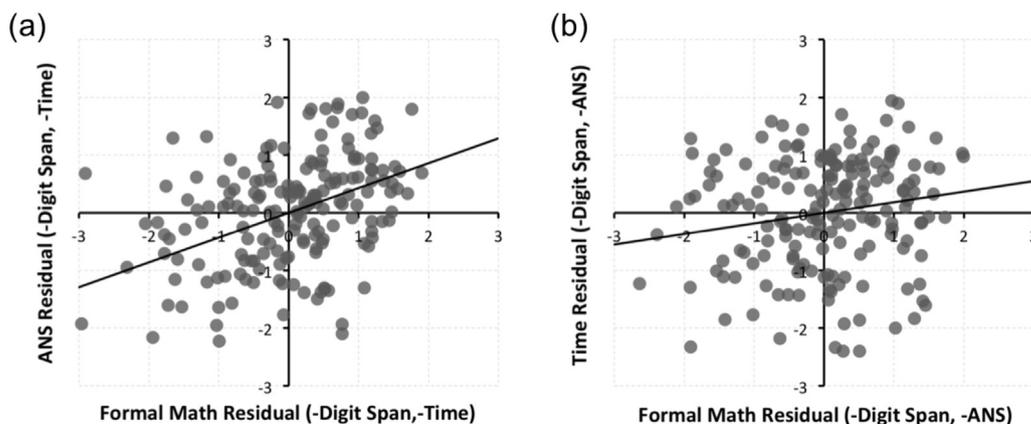


Fig. 6. (a) Partial correlation between the ANS and the Formal Math Score (controlling for digit span and time); (b) Partial correlation between the time and the Formal Math Score (controlling for digit span and ANS). The units are standardized residuals in both.

suggest that approximate magnitude representations encompass a constellation of abilities rather than a single system. That is, while time and number discrimination are related, and they each relate to school math ability, these relations are varied and textured rather than identical or reducible to a single construct. This is consistent with other findings in the literature that have found differences in the neural localization for time and number (Dormal, Andres, & Pesenti, 2008), and findings that suggest that the interference and mapping effects between time and number are the product of a metaphorical mapping between them, rather than shared resources (Bottini & Casasanto, 2013; Vicario & Martino, 2010). More broadly, they are consistent with recent critiques of the generalized magnitude system, including work dissociating number and surface area (Odic et al., 2013), and work suggesting that many of the commonalities between quantity dimensions may stem from shared decision-making components (Van Opstal et al., 2008; Van Opstal & Verguts, 2013).

Our findings are broadly consistent with previous work on the ANS, non-numeric dimensions and formal math. For example, much like in the case of spatial extent in adults (DeWind & Brannon, 2012; Lourenco et al., 2012), we find that the ANS uniquely correlates with formal math compared to time. Similarly, our results are consistent with the finding that adults with dyscalculia have unimpaired time perception (Cappelletti et al., 2011). Our data also extend this work by demonstrating a dissociation between number and time in children, removing the possibility that this division occur only after years of experience and formal math education.

At the same time, our results are not entirely consistent with some previous work showing that time estimation and production are impaired in children with math learning disabilities (Hurks & Loosbroek, 2012; Vicario et al., 2012). One possibility is that the specific nature of the tasks may be responsible: in time estimation and production tasks, participants are asked to assign a numeric value to a duration, or are given a numeric value and are asked to produce a duration of that length. In both of these situations, children must use their knowledge of numbers to perform a task about time. Hence, if their math learning disability had impacted their number knowledge broadly, then we might expect them to be impaired in any task that used numbers, and numerical magnitudes, as stimuli (Cappelletti et al., 2011; Hurks & Loosbroek, 2012; Vicario & Martino, 2010). Furthermore, Vicario et al. (2012) failed to find a correlation between time performance and tests of formal math in their sample; however, without a positive correlation between formal math and the ANS (which they did not test) it is hard to interpret this null result.

Perhaps surprisingly, we found a robust and independent correlation between formal math and the time discrimination performance, even when ANS discrimination performance was controlled for. Although this correlation was consistently weaker than the correlations with the ANS (across the Formal Math Score and the four individual games), it still remains to be explained: why are formal math and time discrimination performance correlated? One possibility is that this correlation is an artifact of a third factor, such as children's ability to pay attention or their executive functions. This, however, seems unlikely given that time continued to correlate with formal math even when digit span performance was controlled for. Alternatively, one's intuitive sense of time may contribute to only certain math abilities (e.g., perhaps providing an analogy to concepts such as multiplication). Future work could investigate whether temporary modulation of time precision has an impact on formal math performance (temporary modulations of the ANS have been shown to selectively impact subsequent math tasks; Wang et al., under review).

In conclusion, the work presented here demonstrates a unique relationship between formal mathematics, the ANS, and time discrimination performance. This suggests that the commonalities between the ANS and time are not due to shared representations on a domain-general "more/less" scale, but more likely due to shared encoding or decision-making processes; and that time and ANS may each

independently relate to formal math abilities. This helps to clarify the nature of the theorized generalized-magnitude system and has implications for future work on mathematics interventions.

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